OCR
Oxford Cambridge and RSA

Thursday 19 May 2022 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

911990254 7

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2022 [603/0991/X]
DC (NF/CGW) 300618/4
OCR is an exempt Charity

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

© OCR 2022

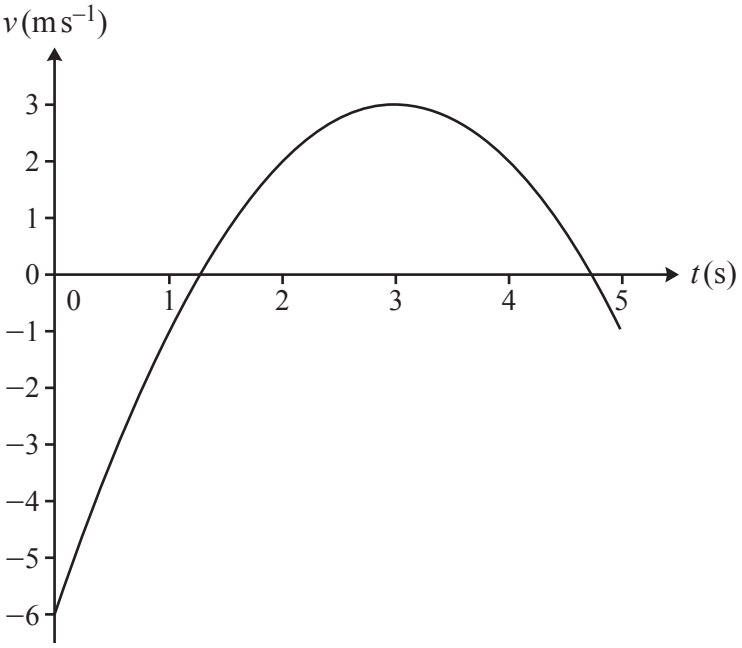
Answer **all** the questions.

1 Rationalise the denominator of the fraction $\frac{2+\sqrt{n}}{3+\sqrt{n}}$, where n is a positive integer. **[3]**

2 **(a)** Determine the value of $\frac{100!}{98!}$. **[2]**

(b) Find the coefficient of x^{98} in the expansion of $(1+x)^{100}$. **[1]**

3 The velocity-time graph for the motion of a particle is shown below. The velocity $v\,\text{m s}^{-1}$ at time $t\,\text{s}$ is given by $v = -t^2 + 6t - 6$ where $0 \leqslant t \leqslant 5$.



(a) Find the times at which the velocity is $2\,\text{m s}^{-1}$. **[2]**

(b) Write down the greatest speed of the particle. **[1]**

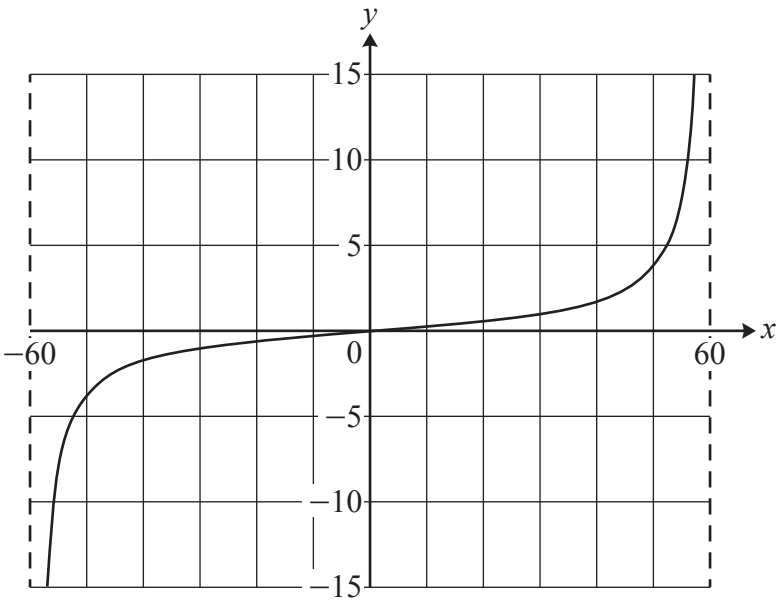
4 The quadratic function $f(x)$ is given by $f(x) = x^2 - 3x + 2$.

(a) Write $f(x)$ in the form $(x+a)^2 + b$, where a and b are constants. **[2]**

(b) Write down the coordinates of the minimum point on the graph of $y = f(x)$. **[2]**

(c) Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = (x+1)^2 - \frac{1}{4}$. **[2]**

5 Part of the graph of $y = f(x)$ is shown below. The graph is the image of $y = \tan x°$ after a stretch in the x-direction.



(a) Find the equation of the graph. **[2]**

(b) Write down the period of the function $f(x)$. **[1]**

(c) **In this question you must show detailed reasoning.**

Find all the roots of the equation $f(x) = 1$ for $0° \leqslant x° \leqslant 360°$. **[3]**

© OCR 2022

6 The gradient of a curve is given by the equation $\frac{dy}{dx} = 6x^2 - 20x + 6$. The curve passes through the point (2, 6).

(a) Find the equation of the curve. **[3]**

(b) Verify that the equation of the curve can be written as $y = 2(x+1)(x-3)^2$. **[2]**

(c) Sketch the curve, indicating the points where the curve meets the axes. **[3]**

7 In this question the unit vectors **i** and **j** are directed east and north respectively.

A canal narrowboat of mass 9 tonnes is pulled by two ropes. The tensions in the ropes are $(450\mathbf{i} + 20\mathbf{j})$N and $(420\mathbf{i} - 20\mathbf{j})$N. The boat experiences a resistance to motion **R** of magnitude 300 N.

(a) Explain what it means to model the boat as a particle. **[1]**

The boat is travelling in a straight line due east.

(b) Find the equation of motion of the boat. **[2]**

(c) Find the acceleration of the boat giving your answer as a vector. **[1]**

8 A team of volunteers donates cakes for sale at a charity stall. The number of cakes that can be sold depends on the price. A model for this is $y = 190 - 70x$, where y cakes can be sold when the price of a cake is £x.

(a) Find how many cakes could be given away for free according to this model. **[1]**

The number of volunteers who are willing to donate cakes goes up as the price goes up. If the cakes sell for £1.20 they will donate 50 cakes, but if they sell for £2.40 they will donate 140 cakes. They use the linear model $y = mx + c$ to relate the number of cakes donated, y, to the price of a cake, £x.

(b) Find the values of the constants m and c for which this linear model fits the two data points. **[2]**

(c) Explain why the model is **not** suitable for very low prices. **[1]**

(d) The team would like to sell all the cakes that they donate.

Find the set of possible prices that the cakes could have to achieve this. **[3]**

9 A tractor of mass 1800 kg uses a towbar to pull a trailer of mass 1000 kg on a level field. The tractor and trailer experience resistances to motion of 1600 N and 800 N respectively. The tractor provides a driving force of 6600 N.

(a) Draw a force diagram showing all the horizontal forces acting on the tractor and trailer. **[2]**

(b) Find the tension in the towbar. **[4]**

10 A triangle has vertices A $(1, 4)$, B $(7, 0)$ and C $(-4, -1)$.

(a) Show that the equation of the line AC is $y = x + 3$. **[2]**

M is the midpoint of AB. The line AC intersects the x-axis at D.

(b) Determine the angle DMA. **[7]**

11 A sports car accelerates along a straight road from rest. After 5 s its velocity is $9\,\text{m s}^{-1}$.

In model A, the acceleration is assumed to be constant.

(a) Calculate the distance travelled by the car in the first 5 seconds according to model A. **[2]**

In model B, the velocity v in m s^{-1} is given by $v = 0.05t^3 + kt$, where t is the time in seconds after the start and k is a constant.

(b) Find the value of k which gives the correct value of v when $t = 5$. **[2]**

(c) Using this value of k in model B, calculate the acceleration of the car when $t = 5$. **[2]**

The car travels 16 m in the first 5 seconds.

(d) Show that model B, with the value of k found in part **(b)**, better fits this information than model A does. **[3]**

12 Below is a faulty argument that appears to show that the gradient of the curve $y = x^2$ at the point (3, 9) is 1.

> Consider the chord joining (3, 9) to the point $(3+h, (3+h)^2)$
>
> The gradient is $\frac{(3+h)^2-9}{h} = \frac{6h+h^2}{h}$
>
> When $h = 0$ the gradient is $\frac{0}{0}$ so the gradient of the curve is 1

(a) Identify a fault in the argument. **[1]**

(b) Write a valid first principles argument leading to the correct value for the gradient at (3, 9). **[3]**

(c) Find the equation of the normal to the curve at the point (3, 9). **[2]**

END OF QUESTION PAPER

© OCR 2022

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.